A Fatigue Design Method for Bolted Joints Subjected to Transverse Vibration

Part 1: A Relationship between Fatigue Strength of A Bolted Joint and Fatigue Strength of A Bolt Alone, And A Fatigue Design Method for Bolted Joints Subjected to Transverse Vibration

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Introduction

Although threaded parts like bolts are one of the oldest mechanical elements, even by now the fatigue failure of threaded parts is not completely gone, and there are quite a few cases where the damage on threaded parts is involved in critical incidents (1). Regarding the fatigue of bolted joint, so far there has been active research on the occasion where the bolted joint is subjected to axial vibration, and tangible fatigue design method is being established (2,3). In VDI2230 (4), a design method is written on the occasion where the bolted joint is subjected to axial vibration, and the one where it is subjected to offset load. On the other hand, there has been much research on self-loosening in terms of when the bolted joint is subjected to transverse vibration, but so far there has been little research on fatigue. However, the threaded parts in mechanical structures lately are more inclined to be subjected to transverse vibration as opposed to axial vibration. Therefore, in recent years the Authors of this article conducted research on fatigue due to the less self-loosening-prone and relatively smaller transverse vibration on the bolted joint, and proposed a fatigue design method for bolted joint subjected to transverse vibration (5-10). This article will explain a fatigue design method for bolted joints subjected to transverse vibration.

Fatigue Strength of A Bolted Joint and Fatigue Strength of A Bolt Alone

Figure 1 shows the bolted joint subjected to transverse load $Q_t$. When a bolted joint is subjected to $Q_t$, there is no slippage between the fastened components; when the fastened components are rigid bodies, load will not exert on the bolt. However in reality, fastened components are elastic bodies, and a slight degree of slippage occurs between them. Therefore, the transverse load $Q_t$ exerting on the bolted joint that deducts the friction force $Q_F$ exerting between the fastened components, as in $(Q_t - Q_F)$, exerts on the bolt through the bearing surface under bolt head. In this occasion, the bolt deforms into an S shape, and depending on the bolt's shape and tightening conditions, where the stress is the highest is mainly the first thread root (namely the main fatigue fracture point in Figure 1). Therefore, this article will explain the fatigue design method for when crack nucleates on the first thread root.

Here, when designing for bolted joints subjected to transverse vibration, it is necessary to calculate the amplitude of transverse vibration load limit within which the nominal stress at the root of the first thread of bolt does not exceed the bolt's fatigue strength. On the other hand, even if the same transverse load exerts on the bolted joint, the nominal stress at the root of the first thread is subjected to slight slippage within the bolt's threaded portion and therefore is dependent upon the tightening condition. Therefore, it is not always easy to derive "the fatigue strength of bolted joint" which is the amplitude of transverse vibration load limit in which the bolt does not fail due to fatigue. In terms of the bolted joint subjected to transverse vibration that the Authors here have been studying, the article will first explain transverse load and the nominal stress at the root of the first thread of bolt as well as their relationship, and then introduce its fatigue design method.

Figure 2 shows the schematic illustration of a deformed bolt due to transverse load. In the schematic illustration of a bolted joint in Figure 2, linear rollers are set between the fastened components to exclude the influence of friction force $Q_F$ between the fastened components, and the transverse load $(Q_t - Q_F)$ is exerted on the upper fastened component. Here, $(Q_t - Q_F)$ is represented by $P_t$. In Figure 2, is the transverse displacement of the fastened object due to $P_t$; $\delta$ is the displacement of the bolt head. $ls$ is the grip length of the bolted joint; $le$ is the engaging thread length of the bolted joint; $la$ is the length of the bolt shank; $lb$ is the thread length which does not engage with the internal threads. Furthermore, the point under the bolt head is named Point O, the boundary between the bolt threaded portion and bolt shank is named Point A, and the point where the bolt starts to engage with the internal threads is named Point B.
When the upper fastened component is subjected to $P_t$, the bolt's bearing surface slips transversely by a mere $\delta_{w-slip}$, and meanwhile inclines clockwise by a mere $\phi_0$ depending on the elastic deformation on the bearing surface of the bolt and fastened component. Additionally, on the bolt's threaded portion, the threads go into elastic deformation and meanwhile transversely slips by a mere $\delta_{w-slip}$ and inclines by a mere $\phi_B$. At this moment, the bending deformation of the bolt's threaded portion is restrained by the internal threaded components, so the bolt deforms into an S shape as in Figure 2.

Here, if we set the bolt's deflection in the load direction as $\delta_{def}$, the displacement $\delta$ of the fastened components can be expressed in the following equation.

$$\delta = \delta_{w-slip} + \delta_{s-slip} + \delta_{def} \quad (1)$$

Here, the displacement $\delta_h$ of the bolt head is $\delta_{s-slip} + \delta_{def}$, and when the transverse slippage on the bolt's threaded surface as well as bearing surface is much smaller than the bolt's deflection $\delta_{def}$, the following equation is tenable.

$$\delta \cong \delta_h \cong \delta_{def} \quad (2)$$

Next, based on the deformation as shown in Figure 2, the deflection $\delta_{def}$ of the bolt can be expressed in the following equation.

$$\delta_{def} = \frac{P_t \cdot I_a \cdot L_a^3}{3 \cdot E \cdot I_a} + \frac{M_B \cdot I_a^2}{2 \cdot E \cdot I_a} + \phi_h \cdot I_a + \left( \frac{P_t \cdot L_a^2}{2 \cdot E \cdot I_a} + \frac{M_B \cdot L_a}{E \cdot I_a} + \phi_h \cdot I_a \right) \cdot L_a + \frac{P_t \cdot I_b^2}{3 \cdot E \cdot I_b} - \frac{M_B \cdot I_b^2}{2 \cdot E \cdot I_b} \quad (3)$$

In Eq.(3), $E$ is the Young's modulus of the bolt; $I_a$ is the moment of inertia of area at the bolt shank portion between Point O and A; $I_b$ is the moment of inertia of area at the threaded portion between Point A and B; $M_B$ is the bending moment restraining the bolt threaded portion at Point B on the root of the first thread. Additionally, $\phi_B$ is the inclination of the bolt head due to $P_t$. A preliminary experiment result shows that $\phi_0$ is proportional to $P_t$, and is expressed in the following equation where the constant of proportionality is set as $K_w=0$.

Here, $C$ is the coefficient which is dependent on $\phi_0$ and $\phi_B$ and which represents restraining the bolt head and the threaded portion. It changes with the rigidity of the bolt and the clamped parts, the tightening conditions, and the lubrication conditions. In other words, in the deformation as shown in Fig.2, if $\phi_0 < \phi_B$ then $C < 0.5$, if $\phi_0 > \phi_B$ then $C > 0.5$, and if $\phi_0 = \phi_B$ then $C = 0.5$. When using a commercial bolt and the common clamped parts appropriately, there must be backlash at the engaging threaded portion, resulting in $\phi_0 < \phi_B$ and $C < 0.5$.

Fatigue Design Method for Bolted Joint Subjected to Transverse Vibration

Figure 3 shows a flow chart for calculating the fatigue strength $(\Delta P/2)_{w}$ of the bolted joint from the fatigue strength of the bolt material. First we calculate axial fatigue strength $\sigma_{aw}$ from the fatigue strength $\sigma_{w0}$ of the smooth specimen of the bolt material using the following equation:

$$\sigma_{aw} = \frac{\sigma_{w0} \cdot (\sigma_T - \sigma_{0.2})}{\beta \cdot (\sigma_T - \sigma_{w0})} \quad (7)$$

In Eq.(7), $\sigma_T$ is the true stress of fracture of bolt material, $\sigma_{0.2}$ is the 0.2 % proof stress of bolt material, and $\beta$ is fatigue notch factor of the bolt. The Eq.(7) here is the axial fatigue strength $\sigma_{aw}$ at the root of the first thread of the bolt alone. Additionally, due to the shakedown at the root of the first thread, the fatigue strength $\sigma_{aw}$ of mid/high strength bolts is not influenced on the clamping force $F$ which exerts as an average stress.

Next, although the axial fatigue of bolt is caused under the zero-tension axial repetitive load, transverse fatigue is caused under the repeating bending moment. Here, the transverse fatigue strength $\sigma_{tw}$ can be calculated with the following equation dividing the axial fatigue strength $\sigma_{aw}$ with correction coefficient $K_L=0.92$ for the loading status.

$$\sigma_{tw} = \frac{\sigma_{aw}}{K_L} \quad (8)$$

Using $\sigma_{tw}$ derived from the above equation, we can calculate the bending moment $M_B$ at the root of the first thread of bolt through the following equation altered from Eq.(6).

$$M_B = C \cdot I_g \cdot P_t \quad (9)$$

With $M_B$ expressed in equation (5), the nominal bending stress $\sigma_t$ at the first thread root of the bolt is expressed in the following equation.

$$\sigma_t = \frac{M_B}{I_b} \cdot \frac{d_3}{2} \quad (10)$$

Here, $d_3$ is the diameter of the bolt's thread root.

When designing the bolted joint subjected to transverse vibration, instead of the fatigue strength $\sigma_{w0}$ of the bolt's first thread root, it is important to know the fatigue strength $(\Delta P/2)_{w}$ of the bolted joint which is the value of load amplitude of transverse vibration that can appear load on the bolted joint. Based on the aforementioned theory, this research will introduce the method to calculate the fatigue strength $(\Delta P/2)_{w}$ of the bolted joint from the fatigue strength $\sigma_{w0}$ of the smooth specimen of the bolt material.
If we already know the relation between $P_t$ and $\varphi_0$, we can derive coefficient $C$. In the research, the Authors measured the displacement of the bolt head during applying $P_t$ statically to the bolted joint, and derived $C$. As for the fatigue strength of the bolted joint derived from Eq.(10), it neglects the friction between the fastened objects as hypothesized in Fig.2. Here, adding the friction between the clamped parts to $C$, we can calculate the final fatigue strength of the bolted joint subjected to transverse vibration.

In Eq.(12), $\mu$ is the friction coefficient between the clamped parts, and $F$ is the clamping force of the bolted joint.

What we know from Eq. (12) is that clamping force $F$ contributes a lot to the assurance of the final fatigue strength of the bolted joint. This article will pause here and later explain the examination of this design method in the upcoming Part 2.

**Conclusion**

Based on the relation between transverse vibration and the nominal stress at the root of the first thread of the bolt, this article introduced the method to derive the fatigue strength of the bolt all the way to the "apparent fatigue strength of the bolted joint" which is the maximum value of transverse vibration that can apply load on the bolted joint without resulting in bolt fatigue failure. The effectiveness of the method introduced here will be explained in Part 2. By understanding the relation between transverse vibration and the nominal stress at the first thread root of the bolt, you will naturally find the key to improving fatigue resistance of the bolted joint subjected to transverse vibration, which is what I would like to call for designers of bolted joints to fully understand.

**Reference**